BENHA UNIVERSITY

ECE 344

Microwave Fundamentals

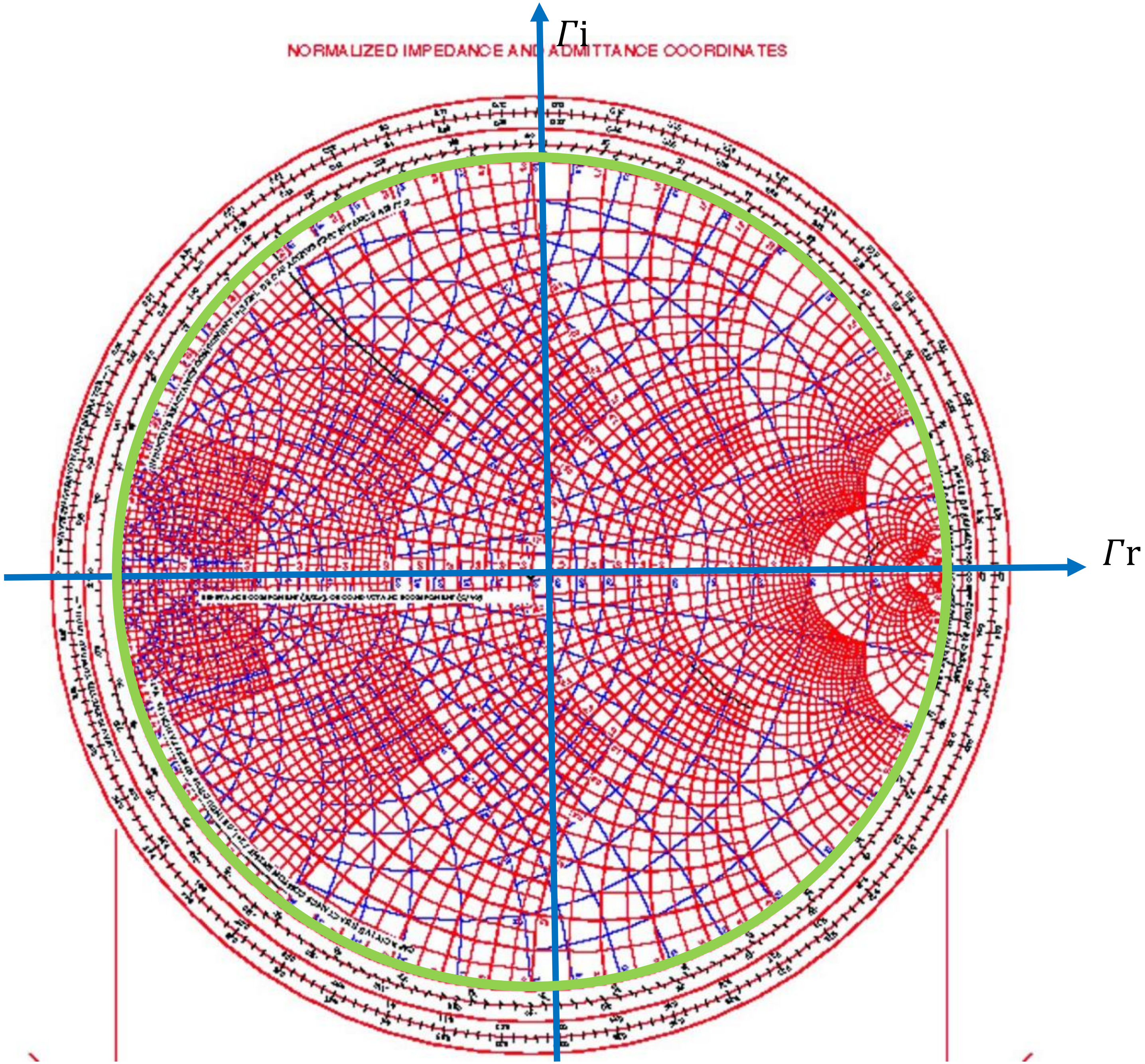
Assistant Professor
Dr. Gehan Sami

Impedance/Admittance smith ch

A chart of Γ

$\Gamma = \Gamma r + j\,\Gamma i$

Max reflection circle
$|\Gamma|$=1



Impedance/Admittance smith chart

You will learn

- Locate impedance on smith chart read corresponding admittance and vise versa, move along TL read corresponding Γin, Zin,VSWR
- Quarter wave transformation
- Adding elements (series-shunt) to load impedance on Smith chart
- Find input impedance to an arbitrary circuit (may contain series, shunt, TL connections)

Impedance (Z) Chart

$$Z(-\ell)=Z_0\left(\frac{1+\Gamma}{1-\Gamma}\right) \qquad \Gamma=\Gamma(-\ell)$$

$$Z_n(-\ell)\equiv\frac{Z(-\ell)}{Z_0}=\left(\frac{1+\Gamma}{1-\Gamma}\right)$$

Define

$$Z_n=R_n+jX_n \;\; ; \;\; \Gamma=\Gamma_R+j\Gamma_I$$

Substitute into above expression for $Z_n(-\ell)$:

$$R_n+jX_n=\left(\frac{1+\left(\Gamma_R+j\Gamma_I\right)}{1-\left(\Gamma_R+j\Gamma_I\right)}\right)$$

Next, multiply both sides by the RHS denominator term and equate real and imaginary parts. Then solve the resulting equations for Γ_R and Γ_I in terms of R_n and X_n. This gives two equations.

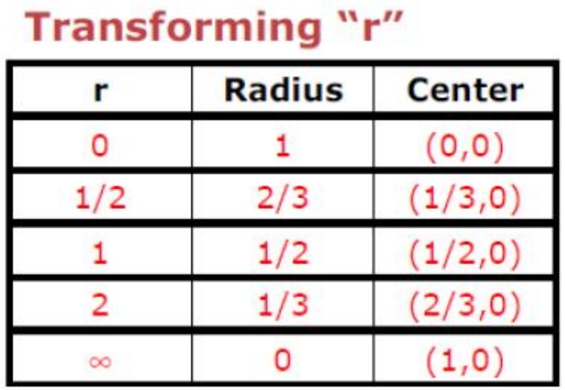
Impedance (Z) Chart (cont.)

1) Equation #1:

$$\left(\Gamma_R - \frac{R_L}{1+R_L}\right)^2 + \Gamma_I^2 = \left(\frac{1}{1+R_L}\right)^2$$

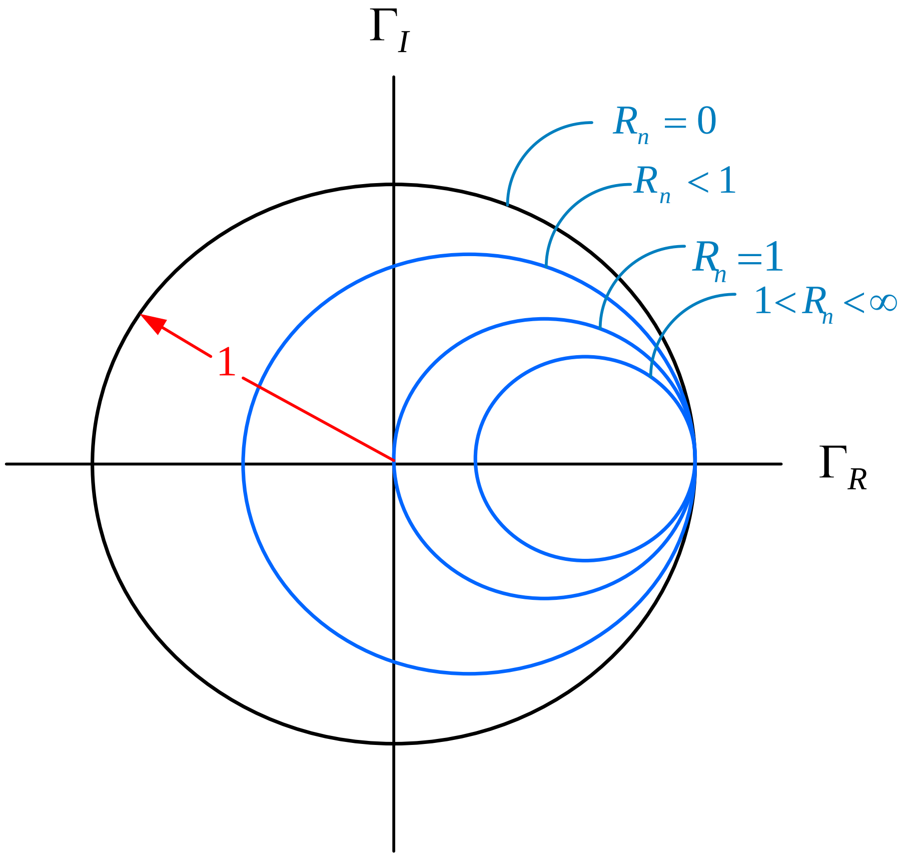
$$\text{center} = \left(\frac{R_L}{1+R_L}, 0\right)$$

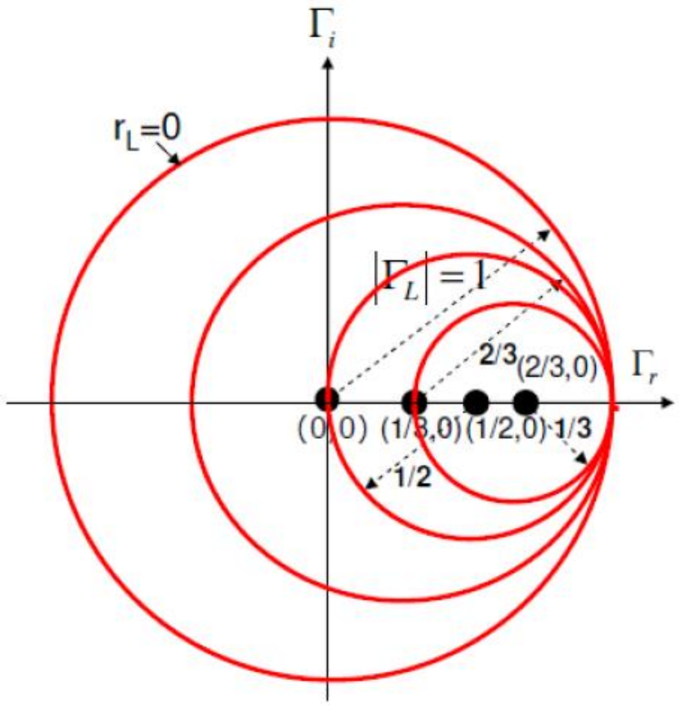
$$\text{radius} = \frac{1}{1+R_L}$$



Transforming "r"

r	Radius	Center
0	1	(0,0)
1/2	2/3	(1/3,0)
1	1/2	(1/2,0)
2	1/3	(2/3,0)
∞	0	(1,0)



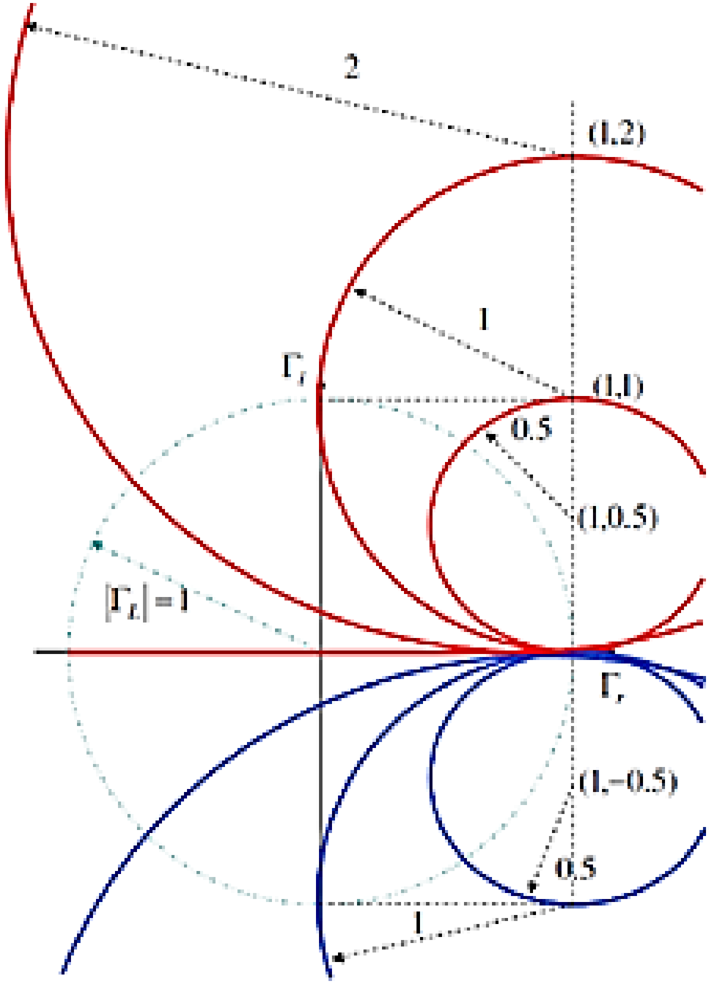


Impedance (*Z*) Chart (cont.)

2) Equation #2:

$$(\Gamma_R - 1)^2 + \left(\Gamma_I - \frac{1}{X_n}\right)^2 = \left(\frac{1}{X_n}\right)^2$$

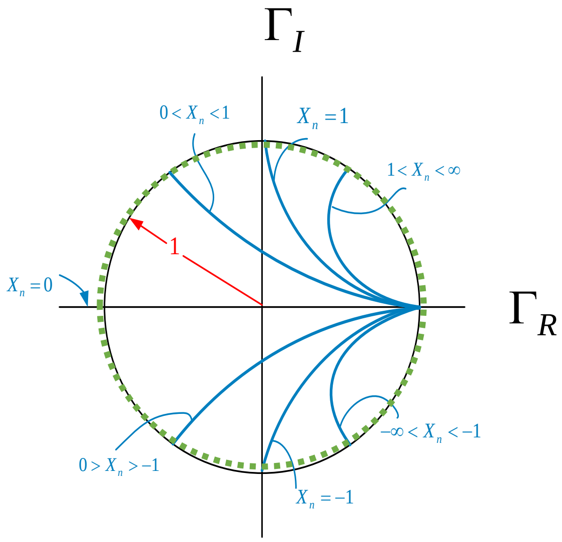
$\text{center} = \left(1, \frac{1}{X_n}\right)$ $\text{radius} = \frac{1}{|X_n|}$



Transforming "x"

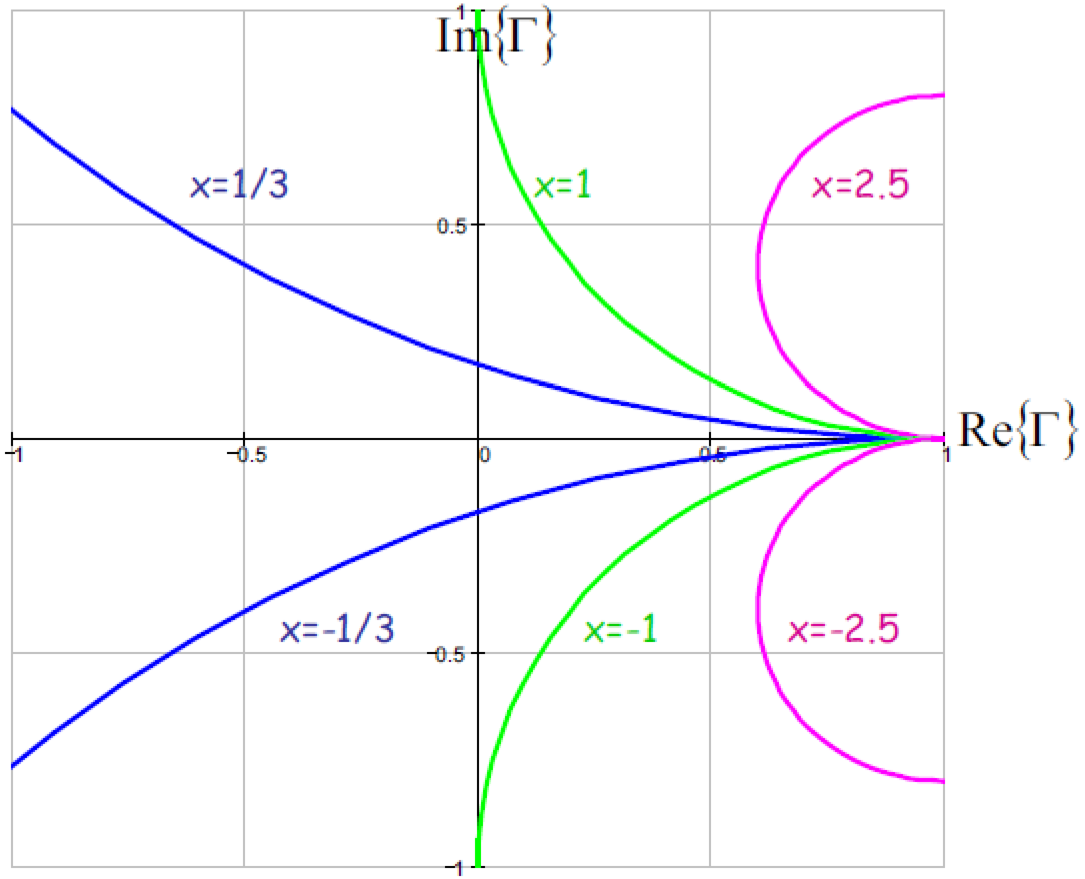
x	Radius	Center
0	∞	(1,∞)
0.5	2	(1,2)
1	1	(1,1)
2	0.5	(1,0.5)
∞	0	(1,0)

x	Radius	Center
0	∞	(1,-∞)
-0.5	2	(1,-2)
-1	1	(1,-1)
-2	0.5	(1,-0.5)
-∞	0	(1,0)

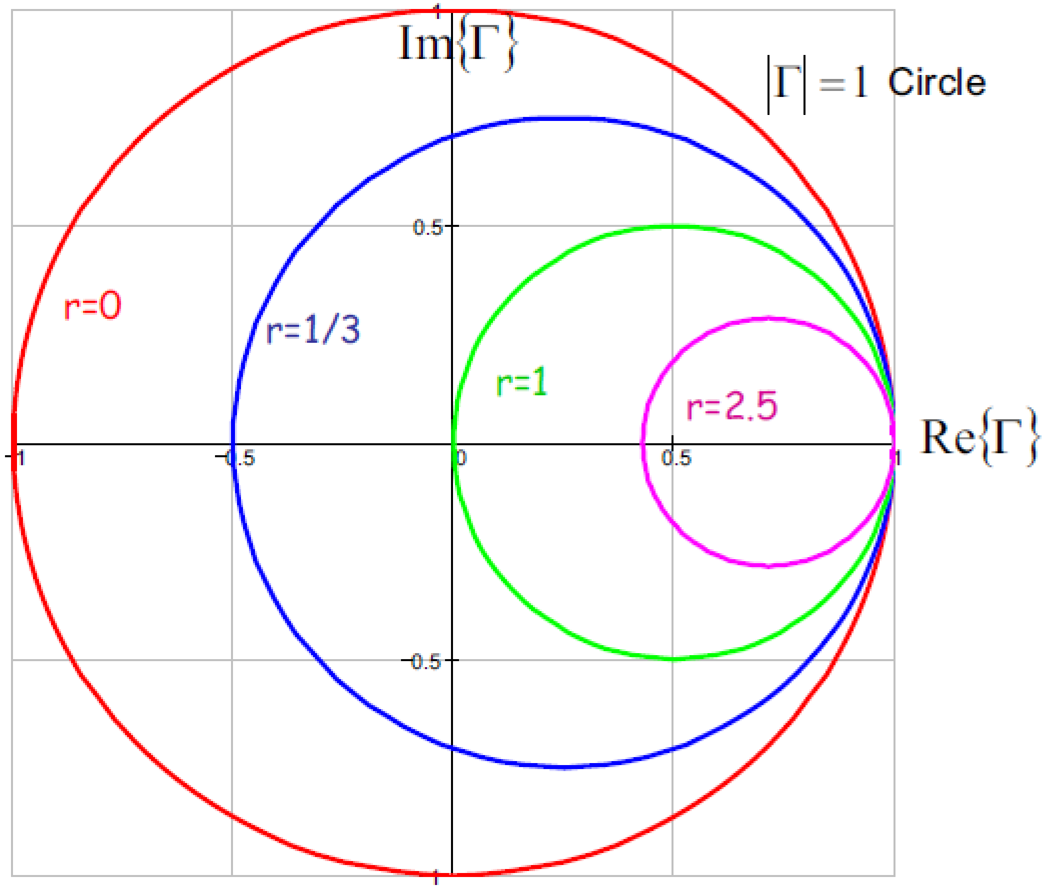


Impedance Smith Chart

Smith Chart – Imaginary Circles



Smith Chart – Real Circles



Impedance (Z) Chart (cont.)

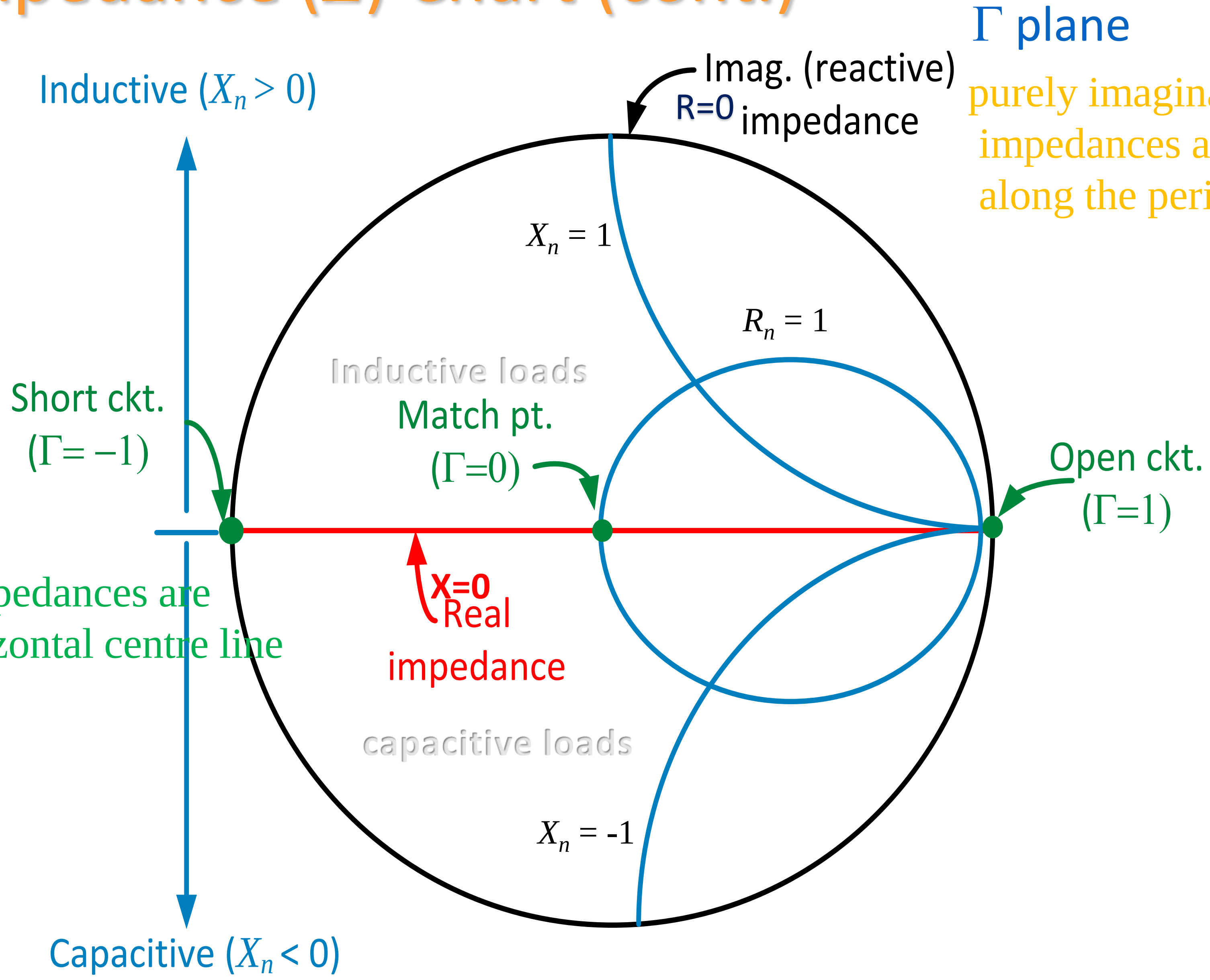
Important Points:

- *Short Circuit*

  $\Gamma = -1,\ z = 0$
- *Open Circuit*

  $\Gamma = 1,\ z \rightarrow \infty$
- *Matched Load*

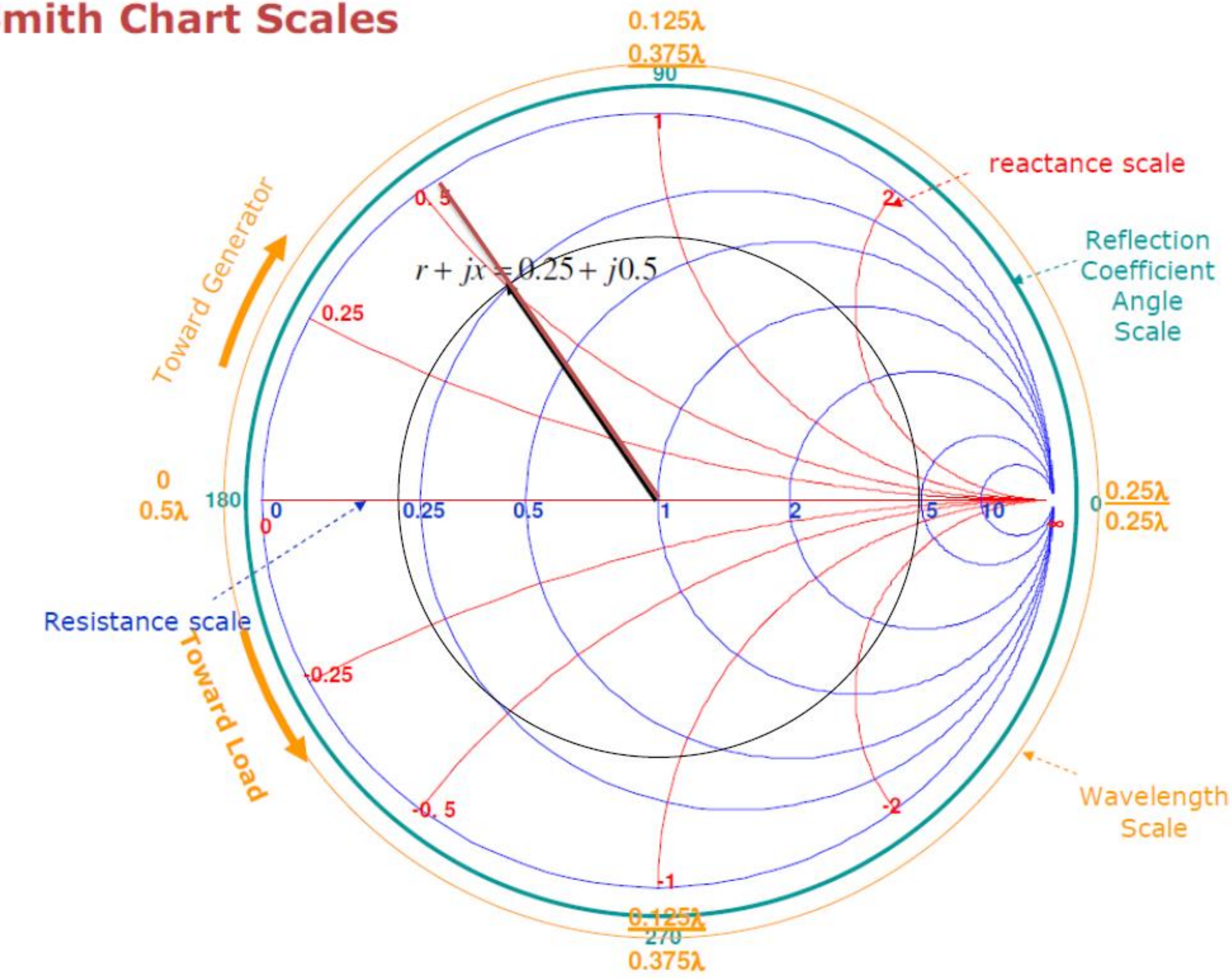
  $\Gamma = 0,\ z = 1$
- *The circle $|\Gamma| = 1$ describes a **lossless element** (C or L)*



purely imaginary impedances are along the periphery

purely real impedances are along the horizontal centre line

Smith Chart Scales



reactance scale

Reflection Coefficient Angle Scale

Toward Generator

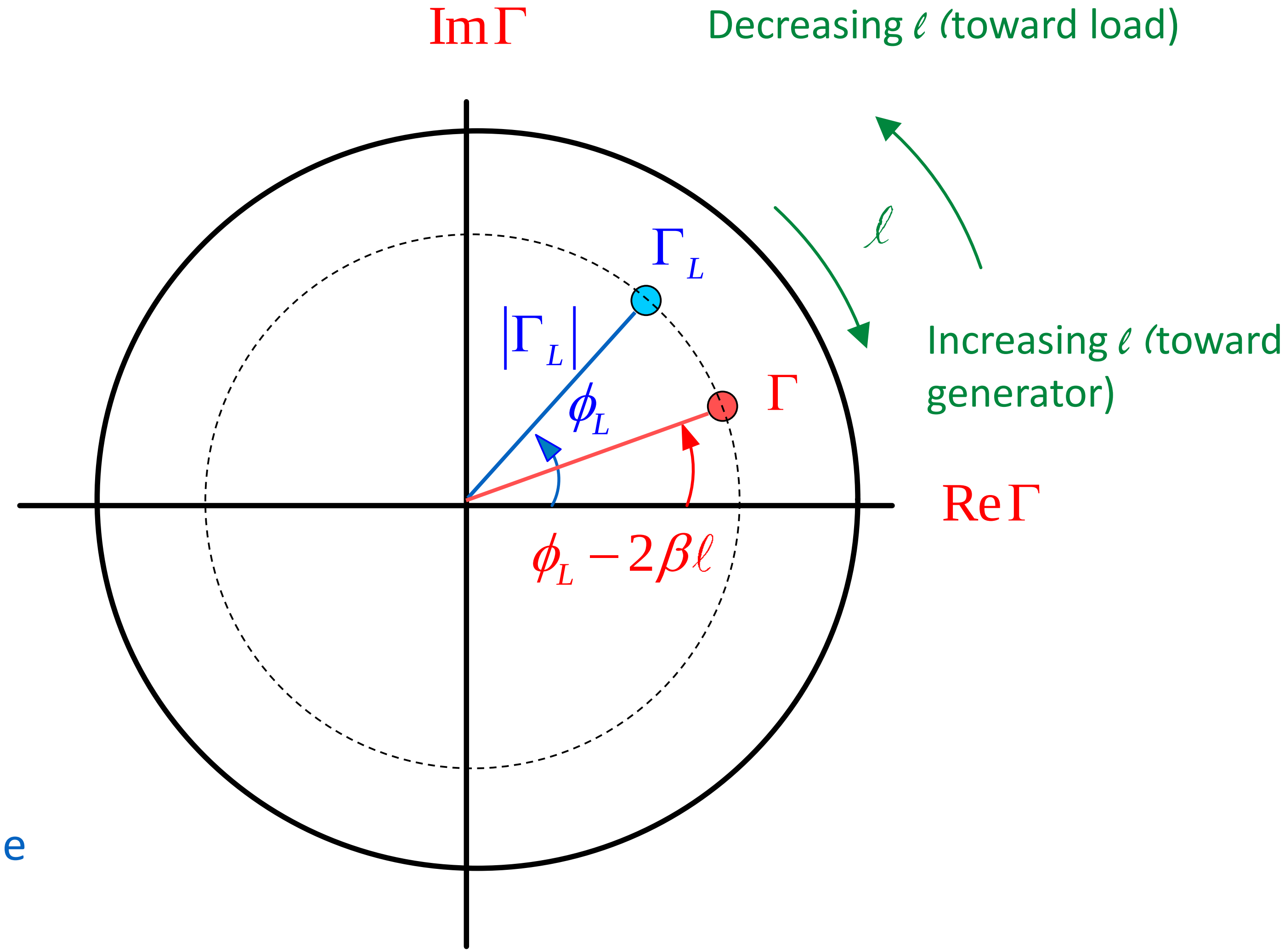
Resistance scale

Toward Load

Wavelength Scale

Complex Γ Plane

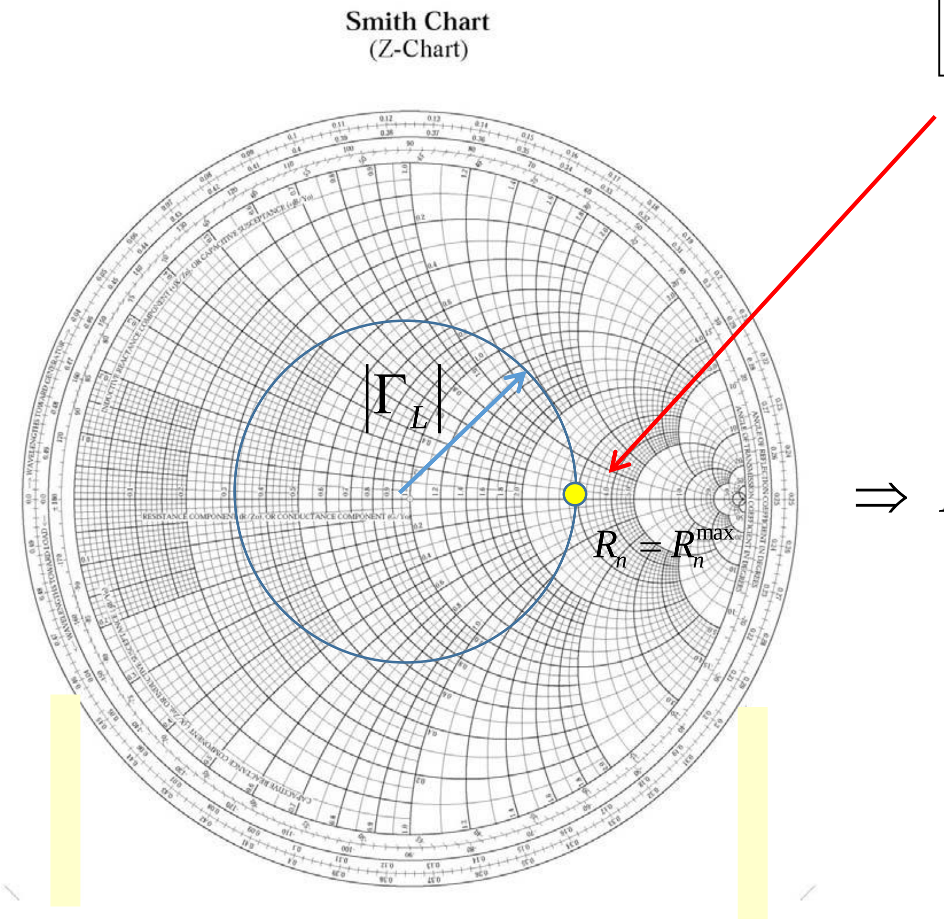
$$
\begin{aligned}
\Gamma &= \Gamma(-\ell) \\
&= \Gamma_R + j\Gamma_I \\
&= \Gamma_L e^{j(-2\beta\ell)} \\
&= |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}
\end{aligned}
$$



$$\Gamma(-\ell) = |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}$$

Lossless line

Standing Wave Ratio


Smith Chart
(Z-Chart)

The SWR is given by the value of R_n on the positive real axis of the Smith chart.

Proof:

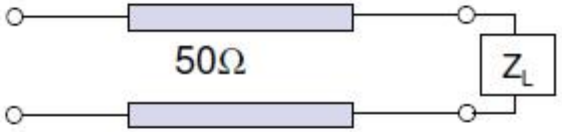
$$SWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$\Rightarrow \quad R_n^{\max} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Example 1

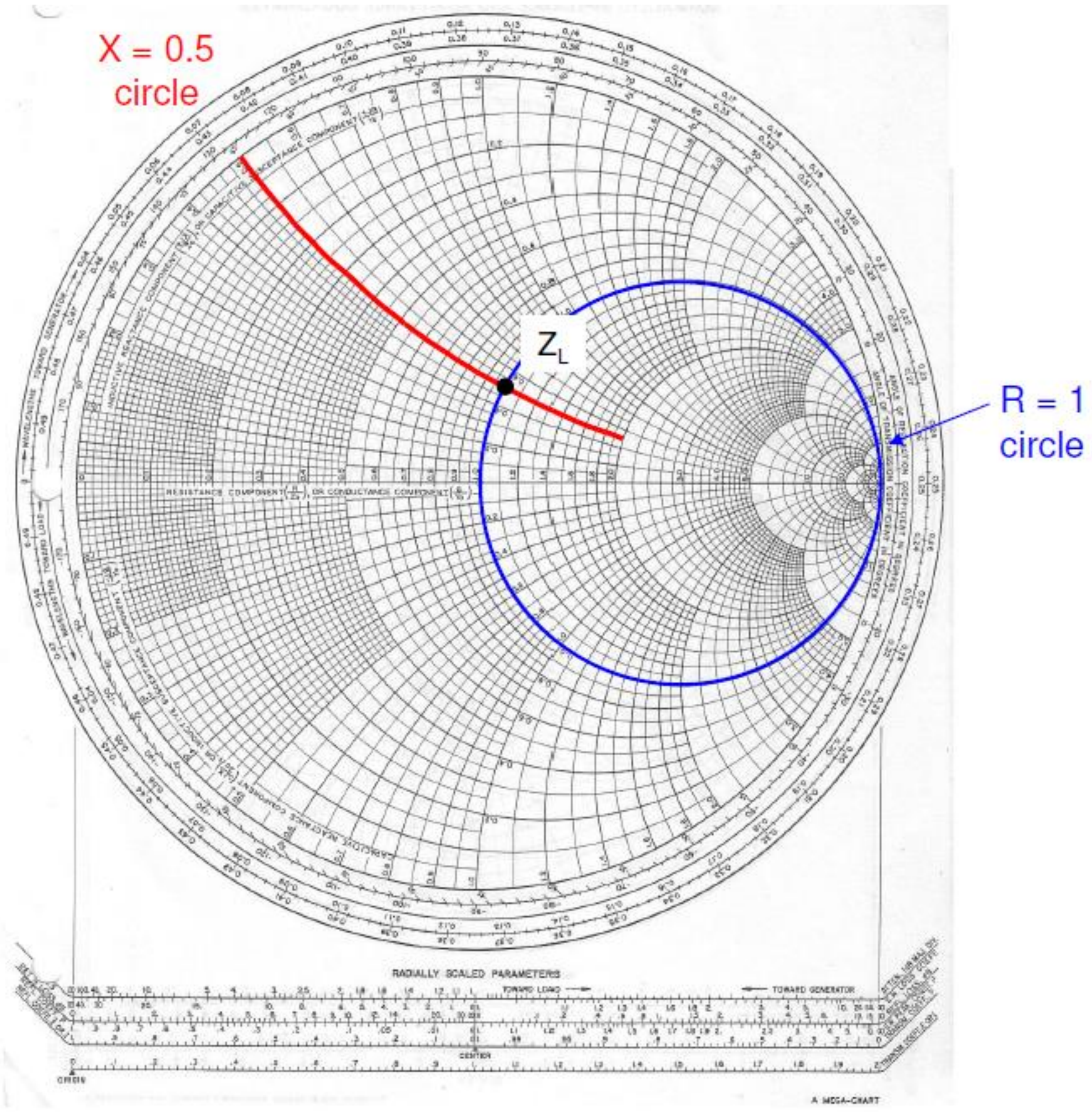
Locate Z

e.g. $Z_L = 50 + j\,25 \ \Omega$



ALWAYS NORMALIZE FIRST

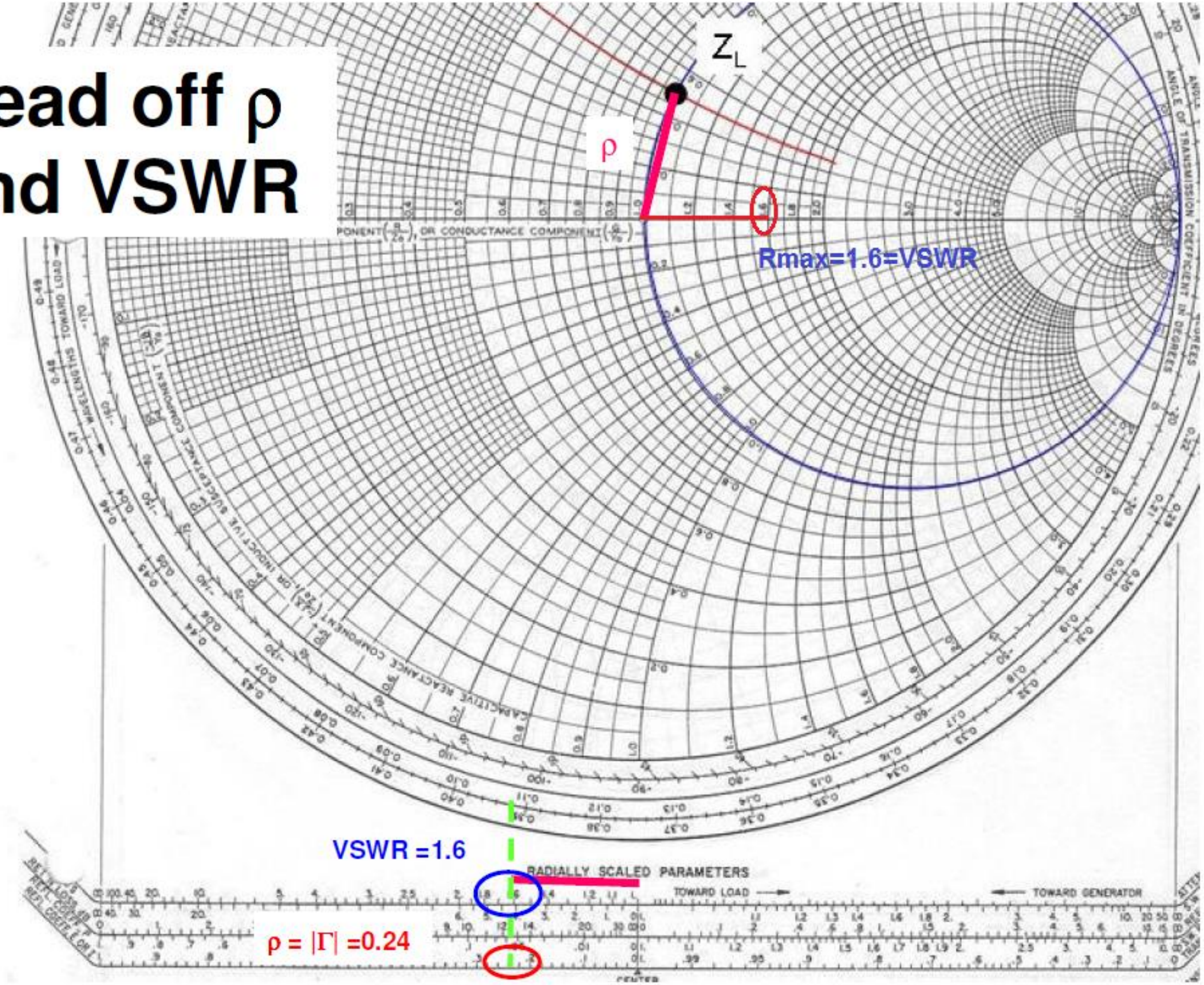
$$\overline{Z}_L = 1 + j0.5$$


X = 0.5 circle

Z_L

R = 1 circle

Read off ρ and VSWR

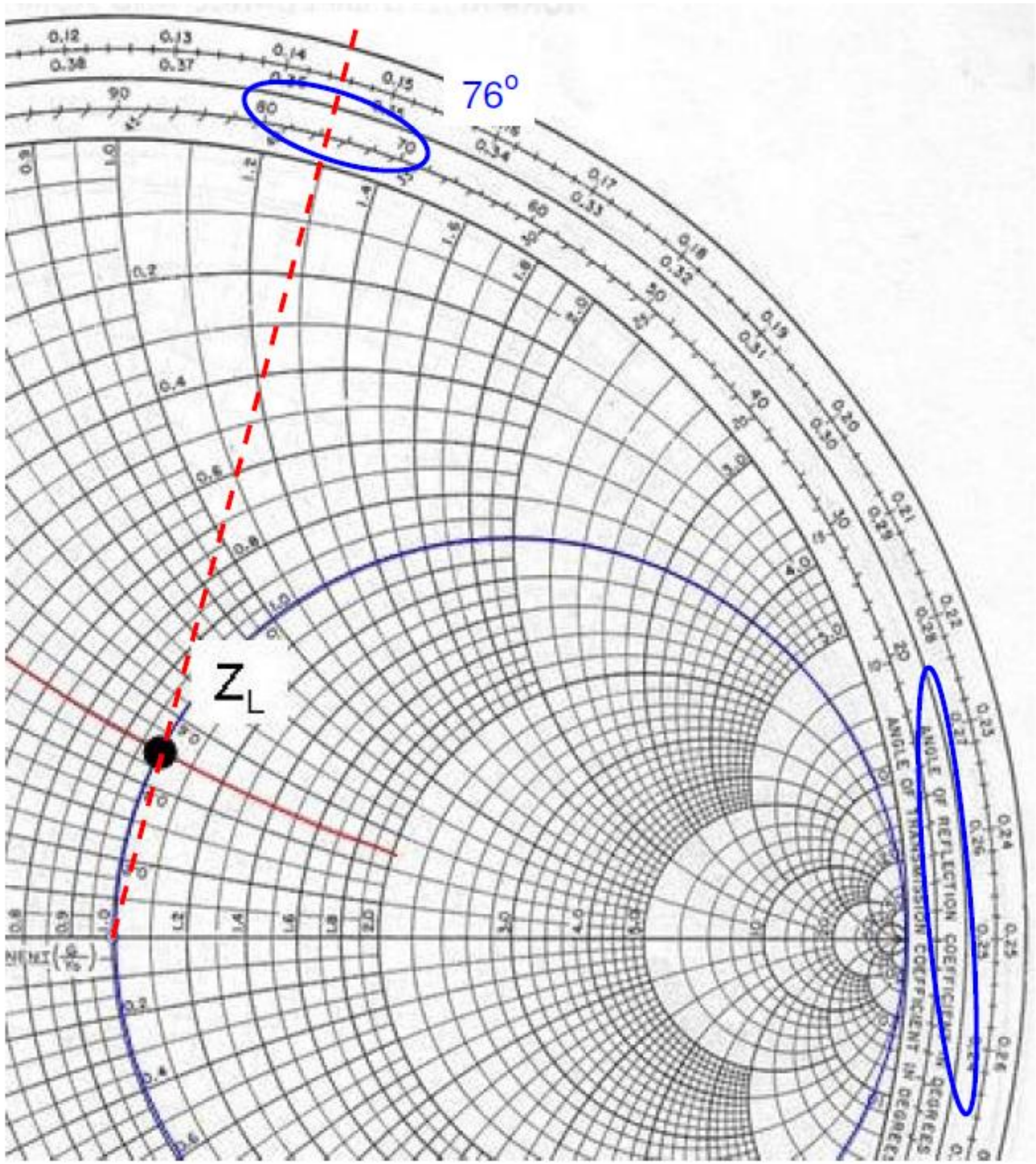


Rmax=1.6=VSWR

VSWR =1.6

$\rho = |\Gamma| = 0.24$

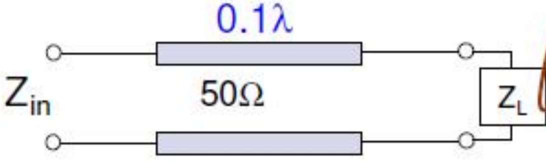
Phase of Γ



$\Gamma = 0.24\ (76^{o})$

Move along TL from load toward Generator

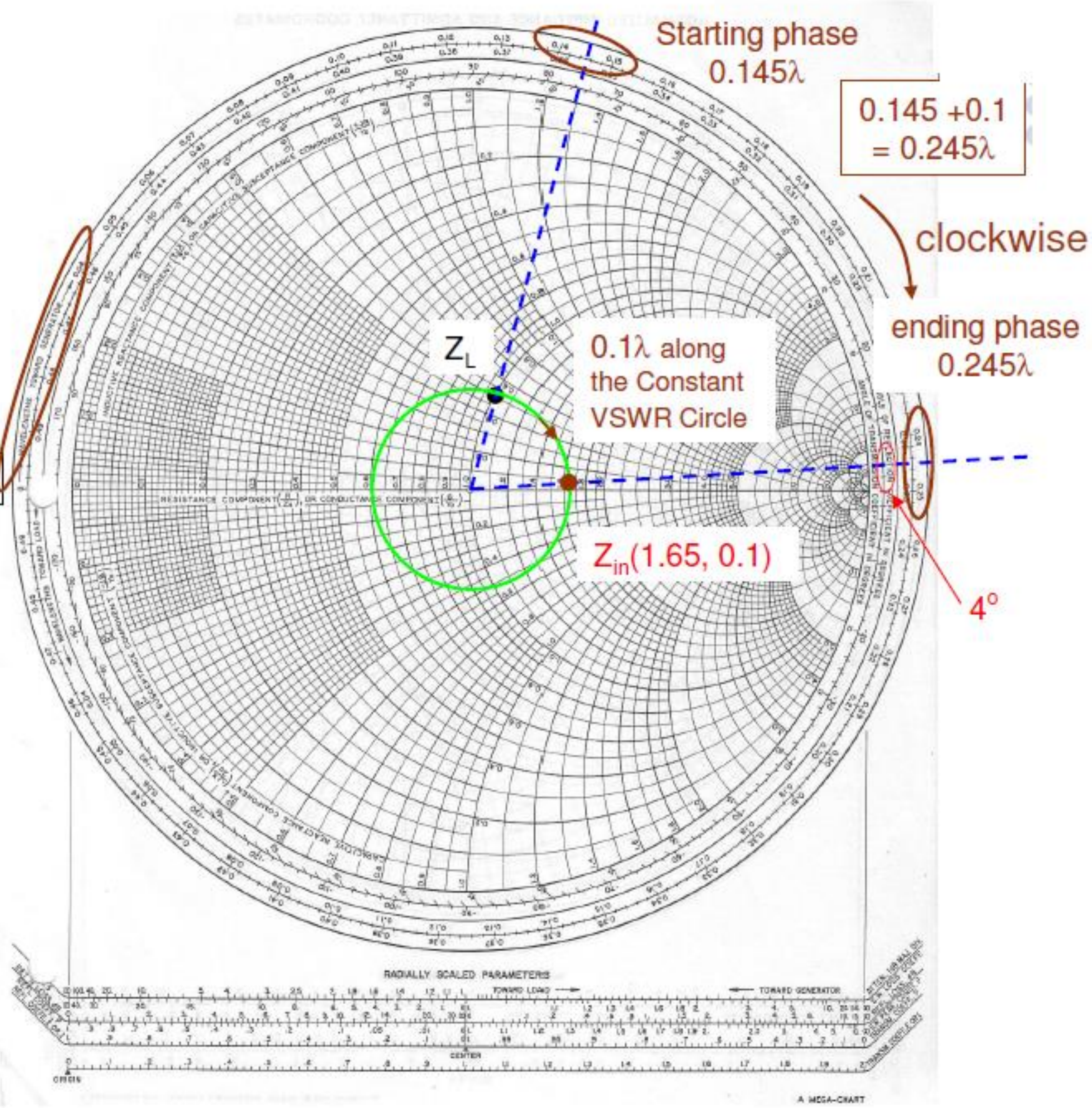
e.g. $Z_L = 50 + j\,25\ \Omega$



All 50 Ω, constant ρ

$Z_{in} = 50(1.65 + j\,0.1)\ \Omega$

$\Gamma_{in} = 0.24\ (4^{\circ})$



Starting phase 0.145λ

0.145 +0.1 = 0.245λ

ending phase 0.245λ

0.1λ along the Constant VSWR Circle

$Z_{in}(1.65,\ 0.1)$